About Ramanujan's Equations

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Abstract:

Equation $x^3+y^3=z^3$ is famous as Fermat's problem that has no natural-number solution . $x^3+y^3=z^3+1$, however, has nutural-number solutions. This equation is a special case of Ramanujan's Equation $x^3+y^3=z^3+w^3$. We used the Maple V program to find out natural-number solutions to $x^3+y^3=z^3+w^3=T$. Only 600 solutions, therefore, are shown below in numerical order. These 600 solutions included a few ones to Case $w=1$.

On the way, we have known a general partial solution to Ramanujan's Equation $x^3+y^3=z^3+w^3$. In addition, we have also known a general partial solution to $x^3+y^3=z^3+1$

From these general solutions, we obtain $x=383662070451$, $y=46411475668533$, $z= 46411484401224$, $w=34878367854$, $T=9997153877261474632492330181409335871928$8 and

$1440000^3+72001^3=1440060^3+1= 2986357263552216001$.

We have shown the two general partial solutions. At the same time, we have obtained numerical value cases, using the general partial solutions and numerical expression processing software, Maple V. And we will discuss about the difficulties involved in searching for all solutions to Ramanujan's Equation.

1. Introduction

We know Fermat Problem（ Equation $x^n+y^n=z^n$ has no natural-number solution when $n>2$）were solved. A similar equation $x^3+y^3=z^3+1$, however, has an infinite number of solutions. Only adding 1 changes the situation of the Equation. This is the mystery of those equations which have an integer solution. We know that Ramanujan's Equation $x^3+y^3=z^3+w^3$ has natural- number solutions. And Condition $w=1$ makes up the equation referred to above.

In this paper, we will show you solutions to Ramanujan's Equation. Those solutions has been found out while using computer programs. To make such programs , it is necessary to take into consideration the multiple accuracy problem. And it is also necessary to take into account the time required to search for the solutions. If x,y,z,and w should vary from 1 to 1000, then, Equation x^3+y^3=z^3+w^3 may be substituted for 1000*1000*1000*1000 times. This is too large to compute entirely. And such status has made it difficult to solve the equation. Shown herein, however, are six hundred sets of the solution to x^3+y^3=z^3+w^3. Those in the past, however, strove to solve such equation using general formulas. As far as Ramanujan's equations are concerned, a general partial solution thereto is to be obtained by correcting the equations known to us. These equations have an infinite number of solutions, but do not cover all the solutions to Ramanujan's equations. The six hundred sets of solutions would turn important.

Anyway, we show numerical-solutions and formulas-solutions to Ramanujan's Equation.

2. The solutions of x^3+y^3=z^3+w^3 (1)

The Basic program, first of all, was used to search for a solution. The Maple V, however, is capable of readily processing the multiple accuracy of an integer. To solve the equation, therefore, the following program was used:

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[> n:=0 : for  x  from 1  to 1000 do
        for  y  from 1  to x do
        for  z  from x+1  to (13/10)*x do
        for  w  from 1  to y do
        if  x^3+y^3=z^3+w^3 then n:=n+1 : print (n,x,y,z,w); fi;
        od;od;od;od;
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we have obtain (n, x, y, z, and w) (1<=n<=600). These solutions are shown in Table 1. And they satisfy x>y, x<z, z>w, and y>w .

To obtain the solutions, 24 hours or more were taken. Using this table 1, we could find out a few solutions, such as x^3+y^3=z^+w^3=s^3+t^3. In other words,

414^3+255^3=423^3+228^3=436^3+167^3

423^3+408^3=460^3+359^3=522^3+111^3

428^3+346^3=492^3+90^3=493^3+11^3

Now, the following formula are available as a general partial solution to equation（1）.

（9*a^7+81*a^4+729*a）^3+（a^9-243*a^3-729）^3=（a^9-729）^3
+（27*a^6+243*a^3）^3

=17496*a^18+531441a^15+4782969*a^12+15943230*a^9+a^27-387420489

a	x	y	z	w
1:	819,	-971,	-728,	270
2:	3906,	-2161,	-217,	3672
3:	28431,	12393,	18954,	26244
4:	171108,	245863,	261415,	126144
5:	757395,	1922021,	1952396,	452250
6:	2628774,	10024479,	10076967,	1312200
7:	7611471,	40269529,	40352878,	3259872
8:	19211976,	134092583,	134216999,	7202304
9:	43584723,	387242613,	387419760,	14526054
10:	90817290,	999756271,	999999271,	27243000

From the large values referred to above, it might be gather that they were outside the range of Table 1. And they were not all the solutions to Equation（1）.

So we can say that they are general partial solutions of Equation（1）.

3. The solutions of Equation x^3+y^3=z^3+1 （２）

Equation （2） is Equation （1） with w=1. The following five solutions were obtained, with Table 1 searched for.

10^3+9^3=12^3+1

94^3+64^3=103^3+1

144^3+73^3=150^3+1

235^3+135^3=249^3+1

438^3+334^3=495^3+1

Euler's solution to Equation（ref 1） x^3+y^3+z^3=1 is:

（-9*t^4-3*t）^3+（9*t^4）^3+（9*t^3+1）^3=1

The first term in the equation above may be transferred to the right side.

Then a general partial solution to Equation x^3+y^3=z^3+1 was obtained. In other words,

$$(9*t^4)^3+(9*t^3+1)^3=(9*t^4+3*t)^3+1$$

This equation was used to obtain the following numerical values:

t	x	y	z
1:	9,	10,	12
2:	144,	73,	150
3:	729,	244,	738
4:	2304,	577,	2316
5:	5625,	1126,	5640
6:	11664,	1945,	11682
7:	21609,	3088,	21630
8:	36864,	4609,	36888
9:	59049,	6562,	59076
10:	90000,	9001,	90030

4 Conclusion

we have shown some solutions to Equatins x^3+y^3=z^3+w^3 and x^3+y^3=z^3+1.

They are not,however, all the solutions to the equations.

Another method, therefore, must be used to search for the solutions not shown.

This is a pleasure for us while showing the importance of Equations x^3+y^3=z^3+w^3 and x^3+y^3=z^3+1 in the areas of algebra and computer sciense.

Reference

1） Andrew Bremner;" INTEGER POINTS ON A SPECIAL CUBIC SURFACE"; DUKE MATHEMATICAL JOURNAL Vol.44,No.4 December 1977