Concomitant Circles of Doval

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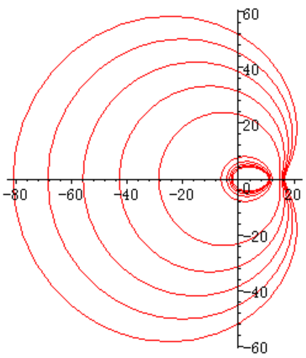


Fig.1Confocal Doval

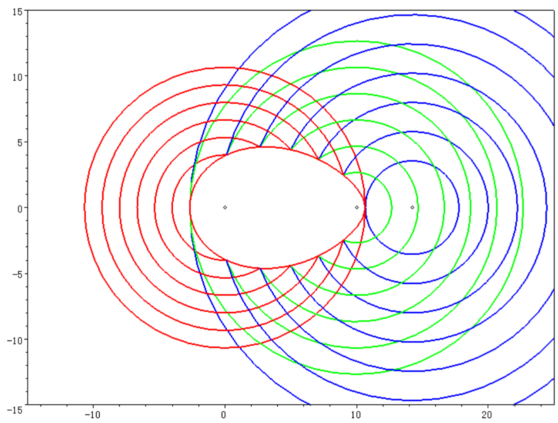


Fig.2　Three focii

$(E_R=0.9, E_L=0.6)$

1　What is Doval

Ellipse is the curve having the same distance from a fixed point and a fixed circle.

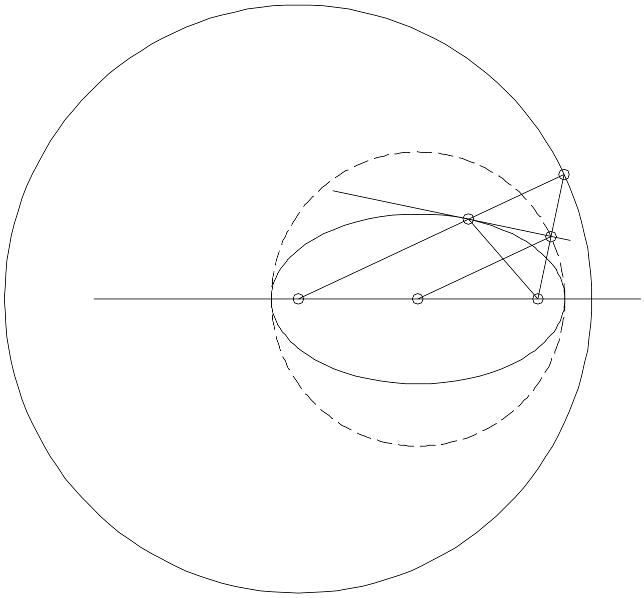
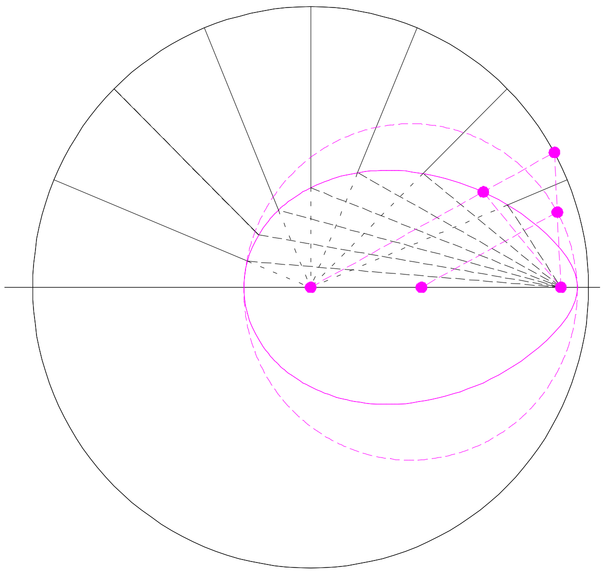


Fig3. Ellipse and its tangent line.

Doval is the curve having constant ratio of two distances from a fixed point and a fixed circle.



FIg。4 Inner part of Doval extended from Ellipse

2 Definition of Doval

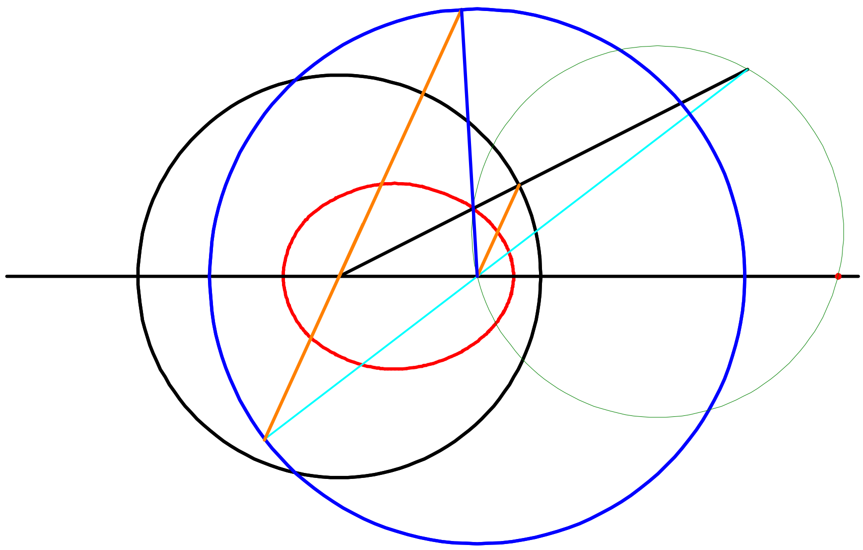
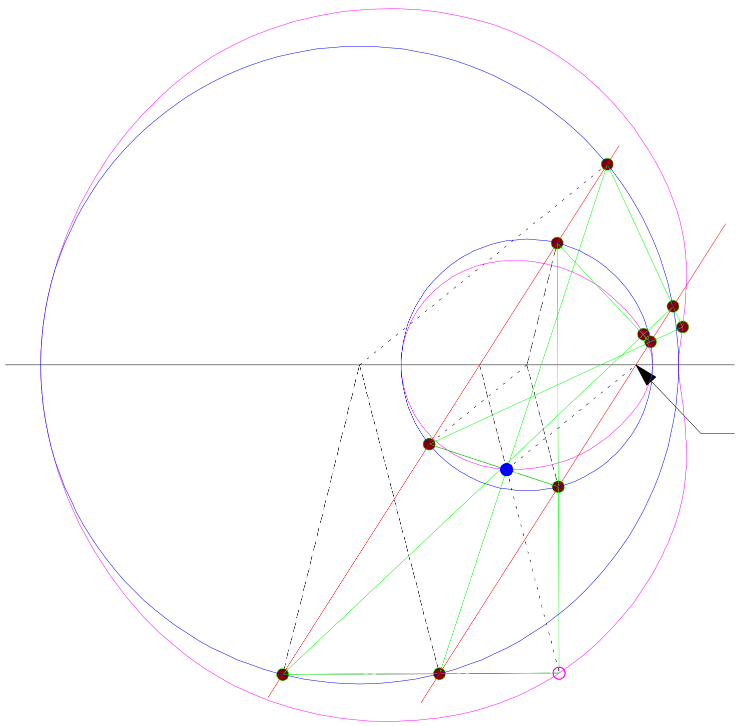


Fig.5 Definition of Doval using two director circles



2つの補助円による卵形線

Fig.6 Definition of Doval using two auxiliary circles

3 Properties on Doval

1 ----Standard Form of Doval Equation--

$mr_1 \pm nr_2 = kc$ is converted to following equation

$$(m^2-n^2)^2\{y^2+X^2-(\frac{k^2m^2+k^2n^2+m^2n^2}{(m^2-n^2)^2})c^2\}^2$$

$$=-\frac{8k^2m^2n^2c^3}{m^2-n^2}X+\frac{4k^2m^2n^2(k^2+m^2+n^2)c^4}{(m^2-n^2)^2}$$

$$X=x+\frac{n^2c}{m^2-n^2}$$

Doval　の　直交定理の証明

(直交定理) 2円の相似中心を通る2直線 (黒線) を引くと、以下の直交点が求まる

(a-b)・(ka+kb)＝k(a・a＋a・b−b・a−b・b) =0

a・a=b・b　なぜなら　a,bともに円の半径の大きさ

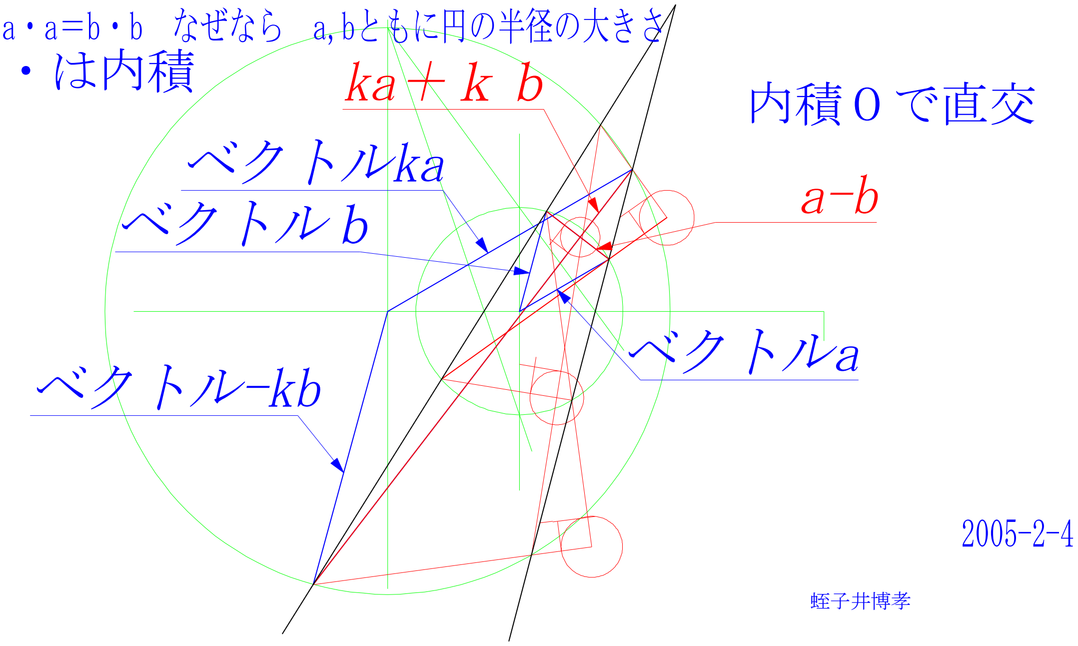
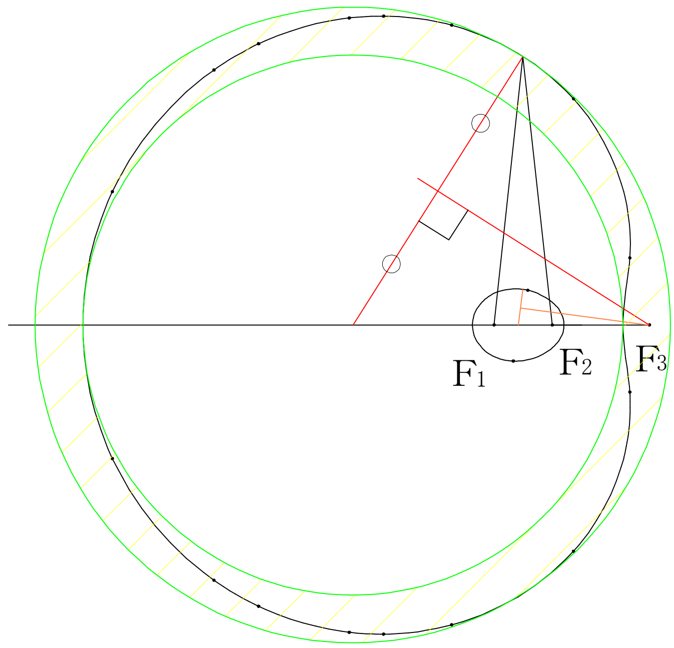


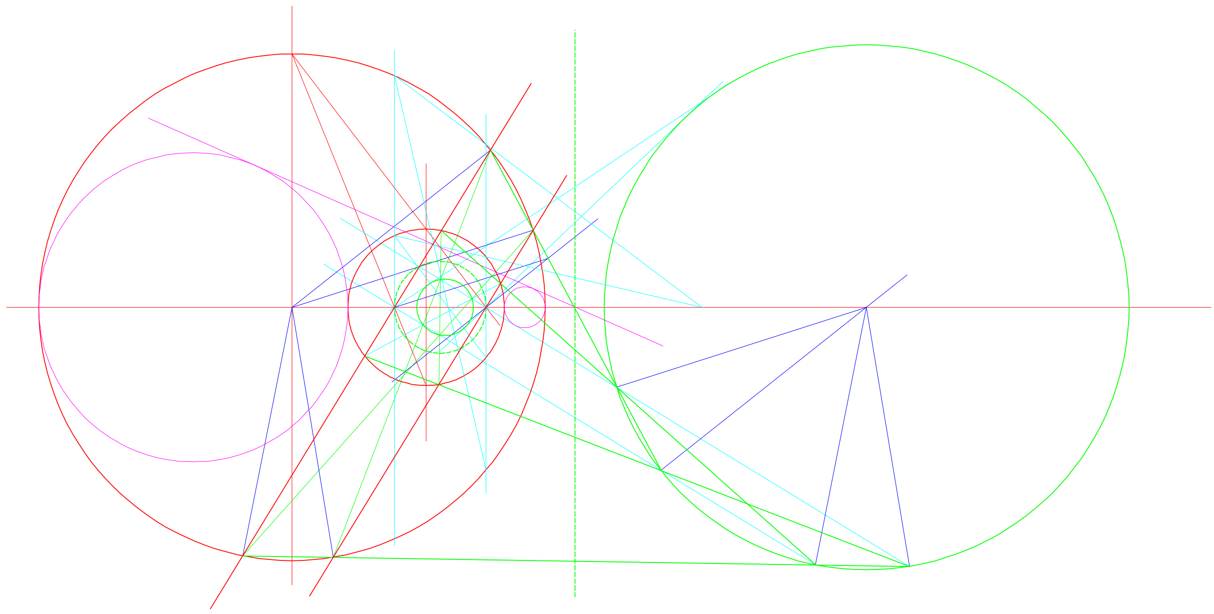
Fig.7　Proof Diagram of Doval Orthogonal Theorem



Perpendicular Bisector of Outer Major Axis Passes F3

Fig.8　Perpendicular Bisectors of Inner Minor Axis and Outer Major Axis Pass through third focus point.

3 Concomitant Circles of Doval(Definition Composition)



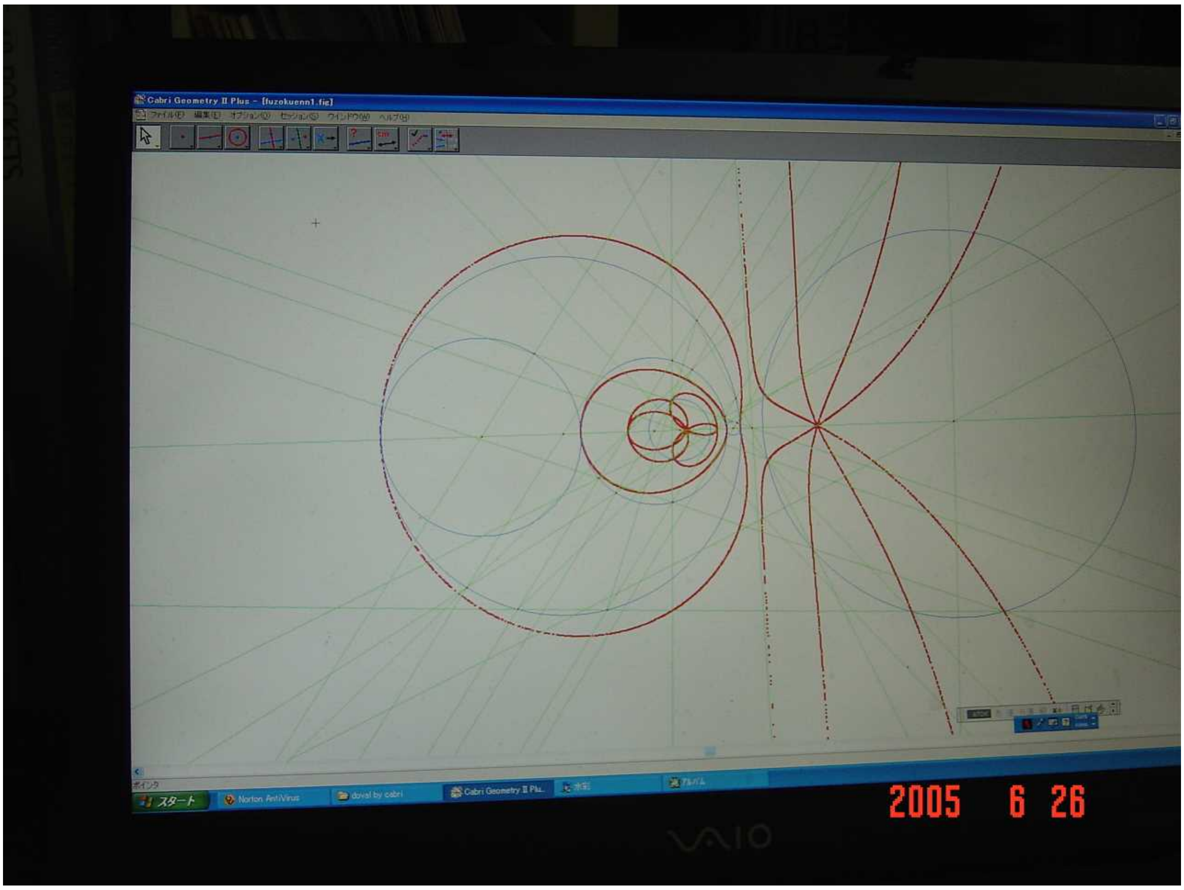
Doval の 随伴円 by H.E

There are two concomitant circles(Green circles) of Doval

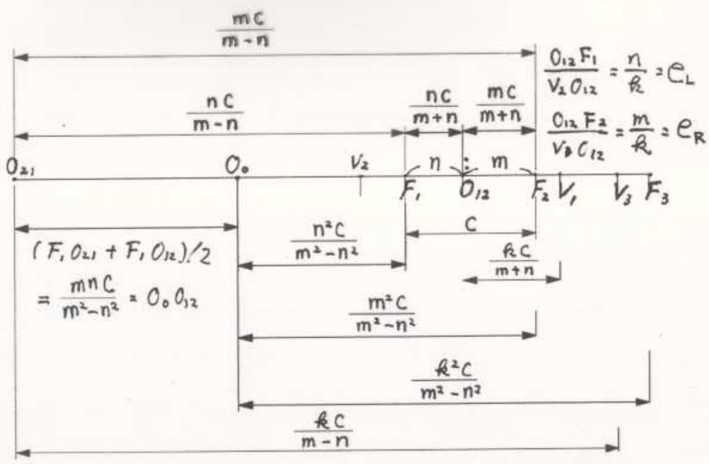
Properties of concomitant circles

1. The centers of concomitant circles move on the line of symmetry axis of Doval.
2. When parallel lines are orthogonal to the symmetry axis,then the radii of the concomitant circles are zero. This position of the concomitant circles is called "Vanishing circle point".
3. The largest diameter of the inner concomitant circle is the same size as the segment which connects the first and second focus points(similarity centers).
4. The diameter of the outer concomitant circle is infinity. The center disappears into the infinity region. The periphery of the outer circle becomes a line which is orthogonal to the symmetry axis of Doval and passes through the three focus point.

4 Concomitant Curves of Doval



3. Distance between Main Points of Doval



文字定数による主要点間の距離

Table 1

*We ashume Doval is defined by $mr_1 \pm nr_2 = kc$

*O_{21}, F_1, O_{12}, F_2 : harmonic range of Points

*O_0 : Middle Point between two CENTERS OF auxiliary Circles (or named Center of equivalent Circles)

*Pairs of these four O_0, F_1, F_2, F_3 on a line define Doval.

Main result of this figure is O_0F_1=n^2C/(m^2-n^2)

O_0F_2=m^2C/(m^-n^2)

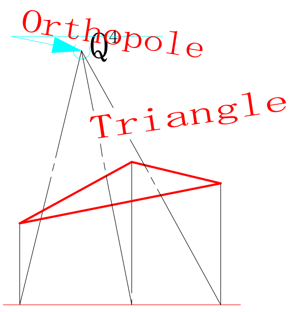
O_0F_3=k^2C/(m^2-n^2)

Raius of three equivalent Circle

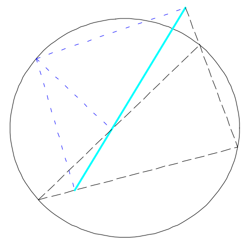
E1=mnC/(m^2−n^2), E2=knC/(m^2-n^2), E3=kmC/(m^2-n^2) BY

5． Infinity Chain Theorem

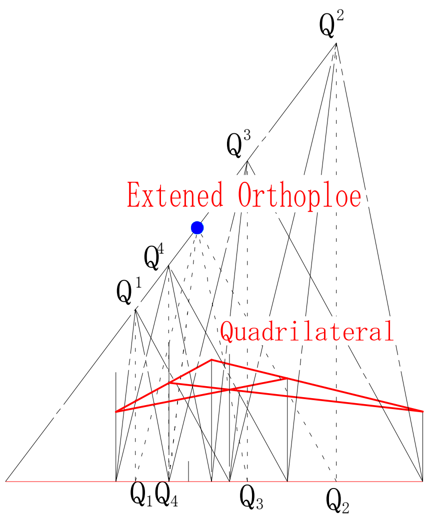
We use following theorem in order to define Chocoid and Tajicoid.



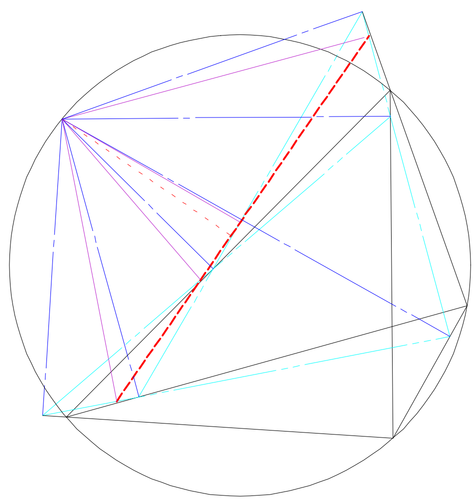
Step 1



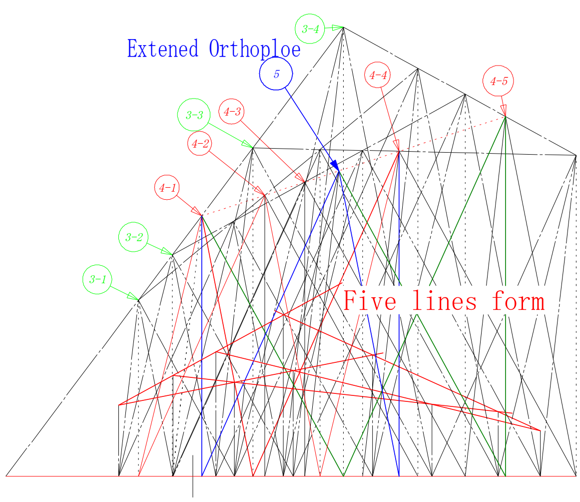
Simson Theorem (Step1(Chain3))

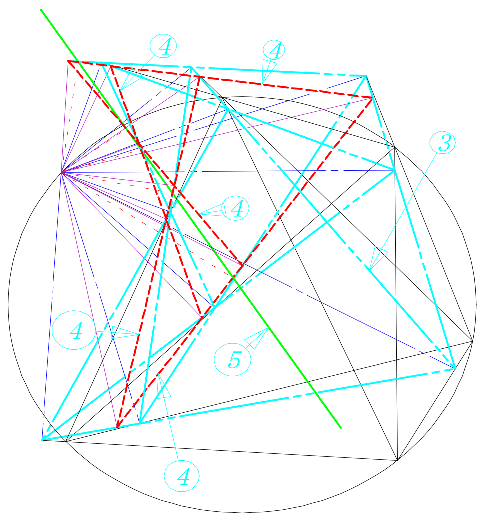


Step 2 (Chain 4)



Step 2(Chain 4)





Step 3 (Chain 5)　　　　　　　　　　　　　　　　　Step 3 (chain 5)

Fig.9. Orthopole Chain　　　　　　　　　　Fig.10. Simson Chain　　by H.E

6． Relation of Extended Curves Chocoid and Tajicoid

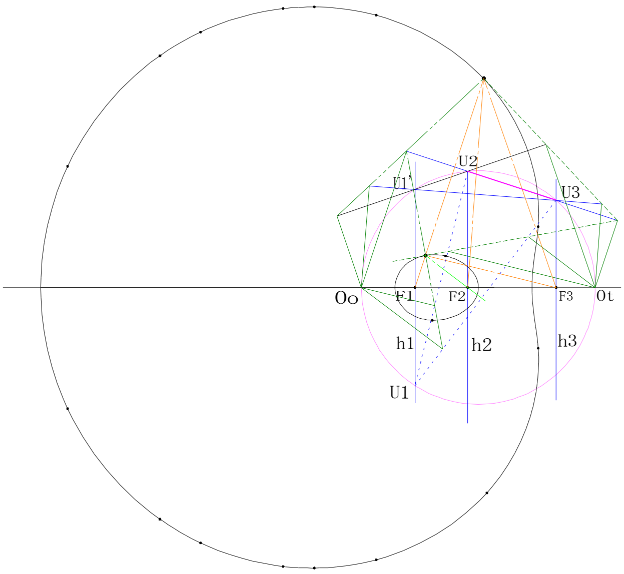


Fig.11.

In this fugure. Orthopole and Simson cross-point are on same position.

(1) Extension of Doval using extended Simson theorem-Composition。

Tajicoid is defined using This figures.
Program is in the proceeding.

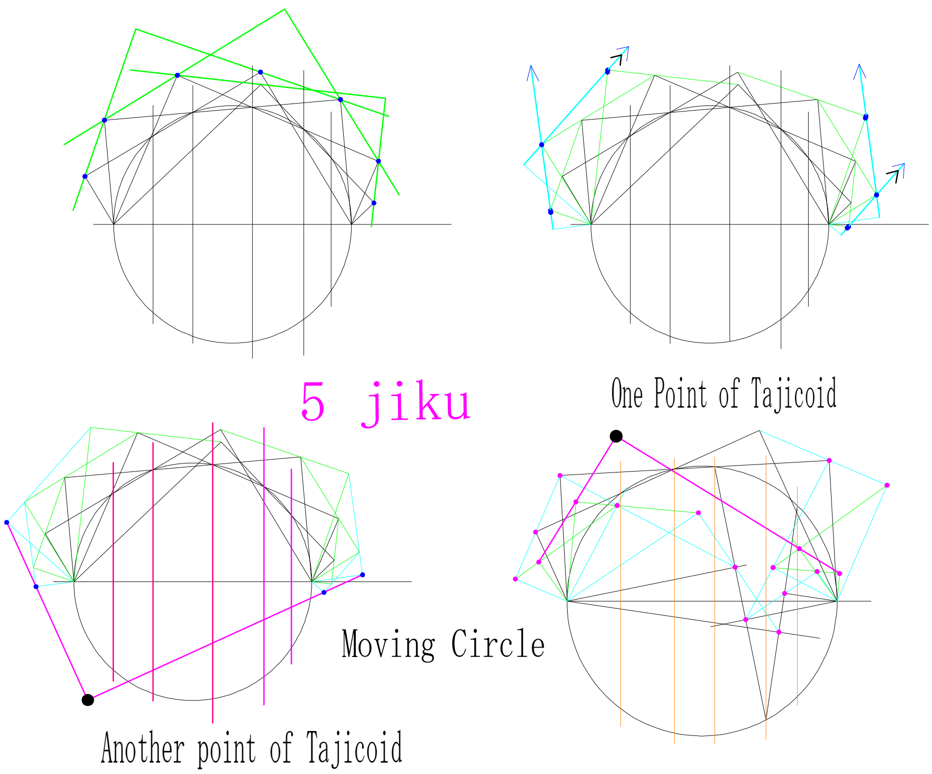
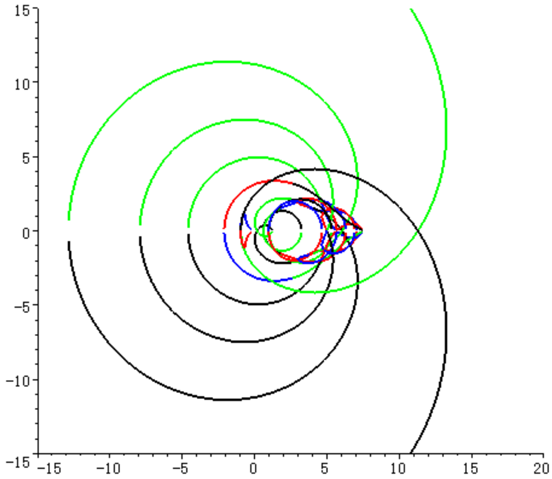


Fig.12.　Def. Figure of Tajicoid　　　　　b y　H.E



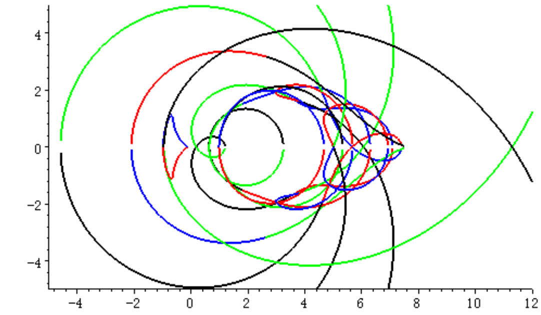


FIG.13.　Tajicoid パラメーター1, 2, 3, 4, 5

(2) Extension of Doval using extended Orthopole theorem-Composition。

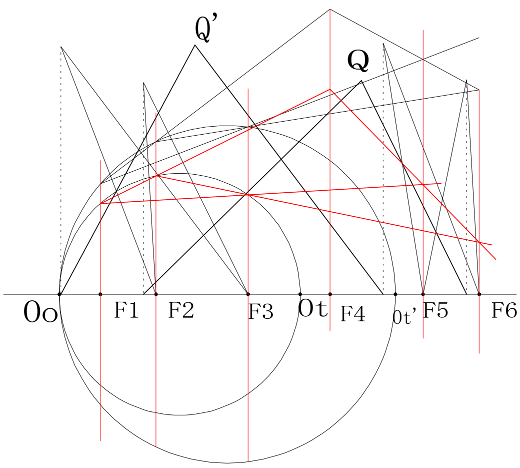
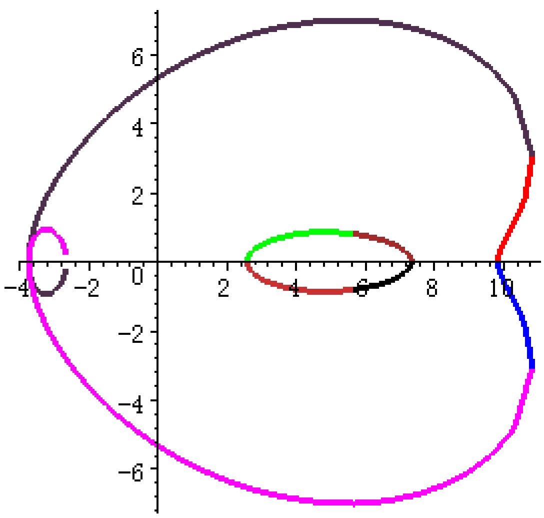


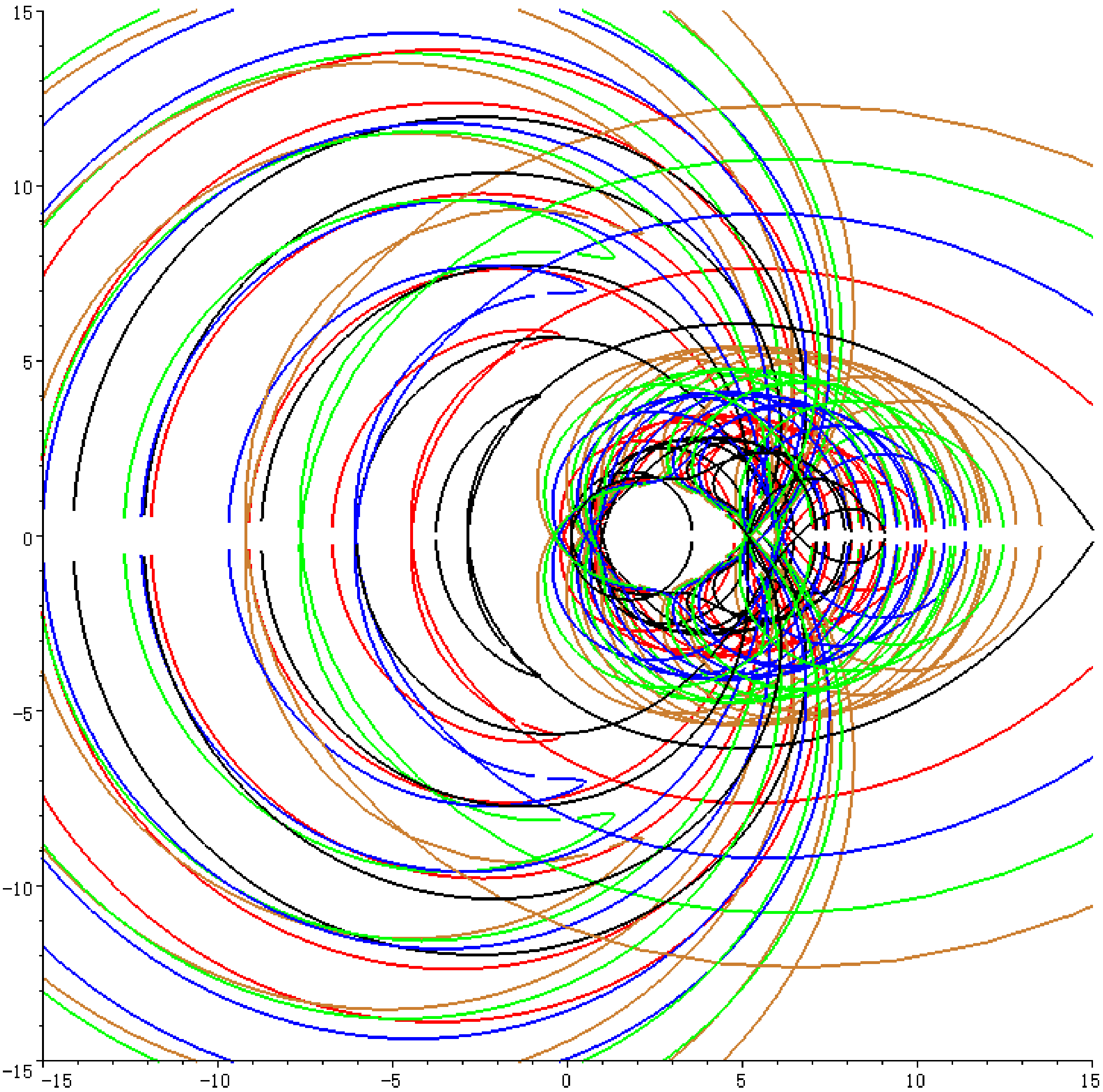
FIG.14.　　DEF Figure Of Chocoid



Parameter x1 = 1, x2 = 2, x3 = 3, x4 = 5, x5 = 150/23, x6 = 165/19

Fig.15.　Chocoid with 6foci　　　by　H.E

7．Confocal Tajicoid



Parameter　**O_0=−1、−2、−3、−4、−5,**

F1～F5= 1. 5, 2, 3, 4, 5

We can draw confocal Tajicoid

because Tajocoid have 5 foci.

Fig.16.　Confocal Tajicoid

By H.E

8 . Conclusion

Today I mainly speak about the Extended Curves.

For extension of Doval, We use Extended Orthopole-Treorem And Extended Simson lines.

Doval has Many properties as writing in proceeding.

But, It is not easy for short time to explain their proof.

So, Today, I intended to show raff sketch how to extend Doval to Extended Curves Tajicoid and Chocoid.

Many Doval propositions exist. And we can feel very fun to find new theorem of Doval.

In the future, we want to find out some applications of Doval.

It might be an application in Mathematics or physics.

Here is Unsolved Probrem of Doval

(1) To find extended conjugate diameter of ellipse.

(2) To find Eccentric angle of Doval like Eliipse

(3) To solve the motion of Oval (Doval) or Ovaloid.

(4) To extend Tajicoid and Chocoid to get Infinity chain of Curves

Anyway, at least, we believe that our research contribute to Curve theorem and to Geometry and CG.

Thanks a lot for your attentions. By H.E