

卵形線を包絡する円群

79-4-22 清書

76-11-26

"デカルトの卵形線の曲率円"の図1において

円(中心 O_0 , 半径 $O_0R_1 = \frac{R_0 C}{m^2 - n^2}$)上に中心 S_1 とす.

半径 R_1, M, ε も円群の方程式で表された

たが O_0 を原点, $\angle R_1 O_0 S_2 = \theta$ とし, $O_0 S_2$ を x 軸とす

$$\begin{aligned} & \left(x - \frac{R_0 C}{m^2 - n^2} \cos \theta\right)^2 + \left(y - \frac{R_0 C}{m^2 - n^2} \sin \theta\right)^2 \\ &= \frac{R^2 n^2}{(m^2 - n^2)^2} C^2 + \frac{m^2 n^2}{(m^2 - n^2)^2} C^2 - \frac{2 R m n^2}{(m^2 - n^2)^2} C^2 \cos \theta. \quad \dots (1) \end{aligned}$$

$$\frac{R_0 C}{m^2 - n^2} = N \quad \frac{R_0 C}{(m^2 - n^2)} = M \quad \frac{m n}{m^2 - n^2} C = K \text{ とおす.}$$

$$(x - N \cos \theta)^2 + (y - N \sin \theta)^2 = M^2 + K^2 - 2KM \cos \theta$$

θ について偏微分する.

$$N(x - N \cos \theta) \sin \theta - N(y - N \sin \theta) \cos \theta = KM \sin \theta$$

$$Nx \sin \theta - Ny \cos \theta = KM \sin \theta$$

$$\begin{cases} (Nx - KM) \sin \theta = Ny \cos \theta \\ x^2 - 2Nx \cos \theta + y^2 - 2Ny \sin \theta = K^2 + M^2 - N^2 - 2KM \cos \theta \end{cases}$$

上式より θ を消去

$$\begin{cases} (Nx - KM) \sin \theta - Ny \cos \theta = 0 & \dots (1) \\ -2Ny \sin \theta + (2KM - 2Nx) \cos \theta = K^2 + M^2 - N^2 - x^2 - y^2 & (2) \end{cases}$$

$$(1) \times (2KM - 2Nx) + (2) \times Ny$$

$$\{-2(Nx - KM)^2 - 2N^2 y^2\} \sin \theta = Ny(K^2 + M^2 - N^2 - x^2 - y^2) \quad \dots (3)$$

$$(1) \times 2Ny + (2) \times (Nx - KM)$$

$$\{-2(KM - Nx)^2 - 2N^2 y^2\} \cos \theta = (Nx - KM)(K^2 + M^2 - N^2 - x^2 - y^2) \quad \dots (4)$$

$$\frac{\{-2(Nx - KM)^2 + N^2 y^2\}^2}{(Nx - KM)^2} = \frac{\{N^2 y^2 + (Nx - KM)^2\}^2 (K^2 + M^2 - N^2 - x^2 - y^2)^2}{(Nx - KM)^2}$$

$$(Nx - KM)^2 + N^2 y^2 = 0$$

$$4\{(Nx - KM)^2 + N^2y^2\} = (x^2 + y^2 - K^2 - M^2 + N^2)^2$$

$x = x + \frac{KM}{N}$ を代入 原点を O_0 として $S_1, 1:5>3$

$$4(N^2x^2 + N^2y^2) = \left\{ \left(x + \frac{KM}{N}\right)^2 + y^2 - K^2 - M^2 + N^2 \right\}^2$$

$$4N^2(x^2 + y^2) = \left\{ \left(x + \frac{KM}{N}\right)^2 + y^2 - K^2 - M^2 + N^2 \right\}^2$$

$$\frac{KM}{N} = \frac{n^2c}{m^2 - n^2}$$

$$\therefore 4 \frac{k^2 m^2 c^2}{(m^2 - n^2)^2} (x^2 + y^2) = \left\{ \left(x + \frac{n^2 c}{m^2 - n^2}\right)^2 + y^2 + \frac{k^2 (m^2 - n^2) c^2 - m^2 n^2 c^2}{(m^2 - n^2)^2} \right\}^2$$

$$4k^2 m^2 c^2 (x^2 + y^2) = \left\{ (m^2 - n^2) \left(x + \frac{n^2 c}{m^2 - n^2}\right)^2 + (m^2 - n^2) y^2 + k^2 c^2 - \frac{m^2 n^2 c^2}{m^2 - n^2} \right\}^2$$

$$4k^2 m^2 c^2 (x^2 + y^2) = \left\{ (m^2 - n^2) x^2 + 2n^2 c x + \frac{n^4 c^2}{m^2 - n^2} - \frac{m^2 n^2 c^2}{m^2 - n^2} + k^2 c^2 \right\}^2$$

$$4k^2 m^2 c^2 (x^2 + y^2) = \left\{ (m^2 - n^2) x^2 + 2n^2 c x + n^2 c^2 + k^2 c^2 \right\}^2 \quad \text{--- A}$$

定義より $m r_1 + n r_2 = k c$ S_1 を原点として

$$m\sqrt{x^2 + y^2} + n\sqrt{(x-c)^2 + y^2} = k c$$

$$\left[m^2(x^2 + y^2) + n^2\{(x-c)^2 + y^2\} - k^2 c^2 \right]^2 = 4m^2 n^2 (x^2 + y^2) \{(x-c)^2 + y^2\}$$

$$m^2(x^2 + y^2)^2 - 2m^2 n^2 \{(x-c)^2 + y^2\} (x^2 + y^2) + n^4 \{(x-c)^2 + y^2\}^2 + k^4 c^4$$

$$- 2k^2 m^2 c^2 (x^2 + y^2) - 2k^2 n^2 c^2 \{(x-c)^2 + y^2\} = 0$$

$$m^4 (x^2 + y^2)^2 + n^4 \{(x-c)^2 + y^2\}^2 + k^4 c^4 - 2m^2 n^2 (x^2 + y^2) \{(x-c)^2 + y^2\}$$

$$- 2k^2 n^2 c^2 \{(x-c)^2 + y^2\} + 2k^2 m^2 c^2 (x^2 + y^2) = k^2 k^2 m^2 c^2 (x^2 + y^2)$$

$$\left\{ m^2(x^2 + y^2) + n^2\{(x-c)^2 + y^2\} + k^2 c^2 \right\}^2 = 4k^2 m^2 c^2 (x^2 + y^2)$$

$$\left\{ (m^2 - n^2) x^2 + 2n^2 c x - n^2 c^2 + (m^2 - n^2) y^2 + k^2 c^2 \right\}^2 = 4k^2 m^2 c^2 (x^2 + y^2) \quad \text{--- B}$$

A, B は一致

ゆえに (1) の包絡線は卵形線である。

R, M, の長さ

$$O_0 O_{12} = \left(\frac{nc}{m+n} + \frac{nc}{m-n} \right) \div 2 = \frac{mn}{m^2-n^2} c.$$

$$S_1 A = \frac{k}{m} c.$$

$$\therefore S_2 A = \sqrt{\frac{k^2}{m^2} c^2 + c^2 - 2 \frac{k}{m} c^2 \cos \theta}$$

$$R, M_1 = S_2 A \times \frac{O_0 O_u}{S_1 S_2}$$

$$= \frac{mn}{m^2-n^2} \sqrt{\frac{k^2}{m^2} c^2 + c^2 - 2 \frac{k}{m} c^2 \cos \theta}$$

$$= \frac{nc}{m^2-n^2} \sqrt{k^2 + n^2 - 2 km \cos \theta}$$