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TWO KINDS (Chocoid, Tajicoid) OF CURVES EXTENDED FROM THE OVAL

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Abstract: In this paper, we describe the Oval of Descartes and its extension. Their composition is defined and expressed using Euclidian geometry and expressed by CAD (Rapid Pro). Ellipse has two focus points. And the oval has three foci. Then, what a curve does it have more foci than three? We have solved this quetion by extended two geometric theorems of the Oval. One extension is defined using extended Orthopoles. Another extension curve is defined using the generalized Simson Theorem. To make their CG ,we use Analytic Geometry . Nowadays, we can draw the orbits of the Oval and its two Extensions using function graphic Software (Maple). Then, first extended curve (named chocoid) is like triple-fold closed curve. On the contrary, second one (named Tajicoid) has strange shapes. We think the reason is inclused in the process of the entension of Simson lines. Anyway, we show their CGs. And at last, we hope this research helps the construction of curve theory and the discovery of some applications of the Oval and its Extension in Physics and Cosmology.
key words: Oval, Orthopole, Simson theorem, Tajicoid, focus point, Extension.

1. Preface

Recent development in scientific instruments is surprising. Thinking of something using tools, tools had been papers and pencil put on the desk, and ruler and compass, since the ancient time. They have turned into personal computer and the utilization of soft wares supporting memory, thinking and presentation has become possible. Naturally the object thought has become to undergo a change from simple one to complex one. Therefore dimension has been extended meanwhile and subjects expressed by figure and image in mathematical science have spread from two-dimensional one to the projection of things in space with higher order dimension on two- dimension space. Among them, non-integer dimension called fractal has appeared. On the other hand, in geometrical subject, the extended subject of order compensating with dimension exists. This began presumably when Descartes etc. disclosed it by discovery of letter coordinate formula. And afterwards this lead to a way of extension from circle with algebraic second order and ellipsoid to the oval with fourth order. This extension of order lead to the discovery of group, ring and field which were tools for verifying self-contradictory proposition of saying that generally algebraic equation with order over fifth cannot be solved algebraically. Moreover on the another hand, an important cycloid or trochoid trajectory etc. by circle movement was researched and this lead to development of present printer which is an equipment of drawing figures. Mathematician call the phenomena caused by movement transcendental, but the whole is not be understood yet. Conversely I hear that algebraic figure can be expressed by cam. This cam movement, discovery of Fourier expression of rectangular drawing expressed by only elementary functions such as SIN and COS and elucidation of electromagnetic and transient phenomena, bring out today's scientific instruments.

Under such a situation, the extension of the oval line with fourth order using only analytical coordinate geometry and algebraically mathematical treatment, was tried this time by movement constitution of elementary figures. In addition to this, CG presentation was tried using function graphics obtained from image expression developed by interpolation method. It will be an agenda in future whether this extension should be called being higher order or transcendental or difficult extension using algebraic equation of root mean square. However, it should be stated definitely that this extension is one accompanying subsidiary property of three foci of the oval line and one of elementary structure.

2 Discovery of the definition of the oval line using theorems in elementary geometry

Here the oval line is defined using elementary geometry (Orthopole point and theorem of Simson line).

2.1 Definition of the oval line using orthopole points

[Definition] As shown in fig. 1, different four points were fixed on a straight line g and they were designated as O, F1, F2 and F3 in succession from the left. Now straight lines h1, h2 and h3 perpendicular to a straight line g are drawn through F1, F2 and F3. Next on a straight line, a moving point T is determined and a circle with the diameter of OT is drawn. But the condition is OT ≧ OF3. And intersection points of this circle and straight lines h1, h2, h3 are designated as U1, U2, U3 respectively. Then the Orthopole point of a straight line g related with Δ U1U2U3 lies on the oval line and if a moving point T moves in the right side of F3, the Orthopole point draws the oval line. That is to say, it draws an inside branch and an outside branch of the oval line while O is the center of equally distant circle with F1, F2 and F3 as first focus point, second focus point and third focus point. Here the equally distant circle consists of a set of points equally distant from an inside branch and an outside branch of the oval line as shown in fig. 2. This proof is abbreviated. Again, for fixing U1, U2 and U3, there are totally 8 ways which corresponds to three powers of 2 from two ways in each point.

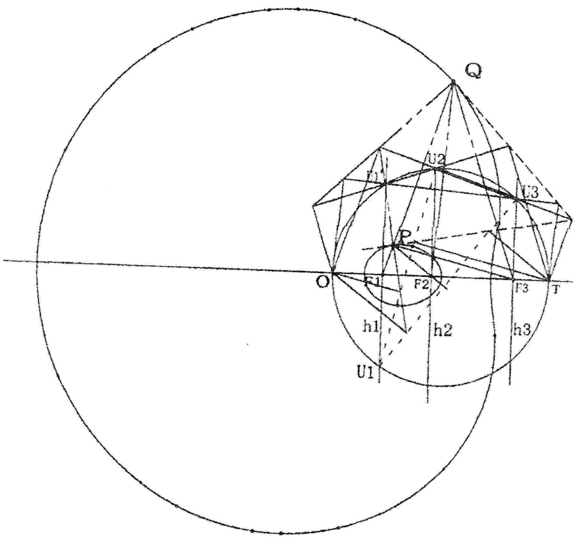


Fig. 1 Orthopole point, Simson lines and oval lines

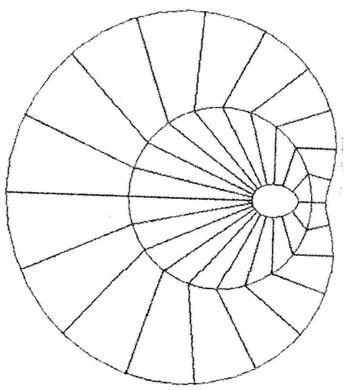


Fig. 2 oval lines and equally distant circle

2.2 Definition using Simson line

[Theorem] As shown in fig. 1, Simson lines formed from both ends O, T of the diameter related with Δ U1U2U3 associated with a straight line g intersect perpendicularly and the intersecting point coincides with the orthpole point of g related with Δ U1U2U3.

This proof is described in [295] of 3 in a large dictionary of geometry edited by Shiyasu Iwata. Accordingly intersection points of Simson lines from two points O and T lies on the oval line and if T moves along g, it draws the oval line.

2.3 Some Theorems of the Oval

[1] As shown in fig. 1, the one of Simson lines intersecting perpendicularly at points of P and Q on the oval line is a normal line and the other is a tangent line.
[2] Intersection points of normal lines of inner branch and outer branch shown in fig. 1 lie on the same circumference and the corresponding circle is called equally distant circle. Refer to fig. 2.
[3] As shown in fig. 3, we obtain the ovals with same focus points. In other words, This figure shows confocal ovals.

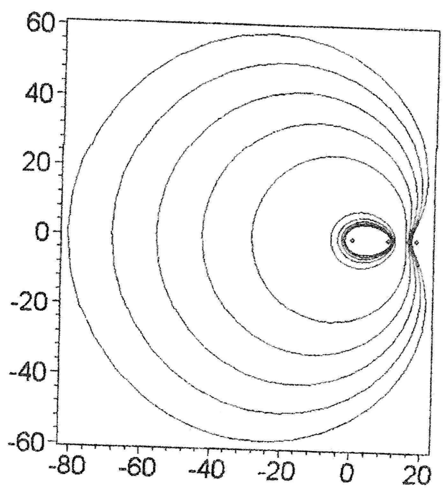


Fig. 3 Confocal Ovals

3. Extension of the oval using generalized Orthopoles(EBISUI, 2001)

First, we show exlended Orthopole theorem in fig. 4. Points 3-1,3-2,3-3,3-4 are Orthopoles of each triangle which can be constructed by three lines among four lines. Next, we draw perpendicular lines to straight line g from each Orthogonal point. Moreover, from each foot point of perpendicular lines, we draw perpendicular lines to the line which remains, when we chose three lines among four lines. Then we can find only one intersection point(④) of the four perpendicular lines. This points is a generalized orthopole.

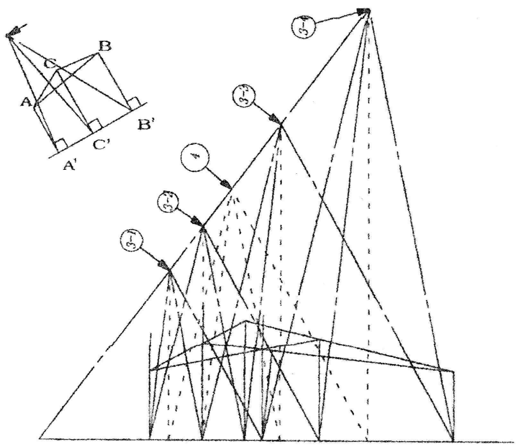


Fig 4 Extened orthopole of four lines

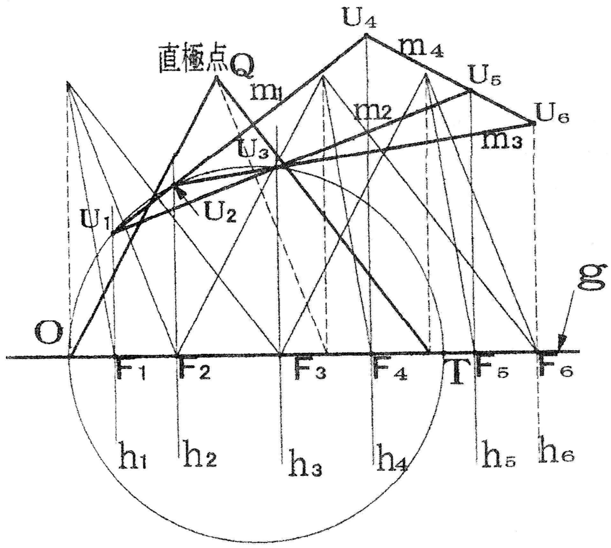
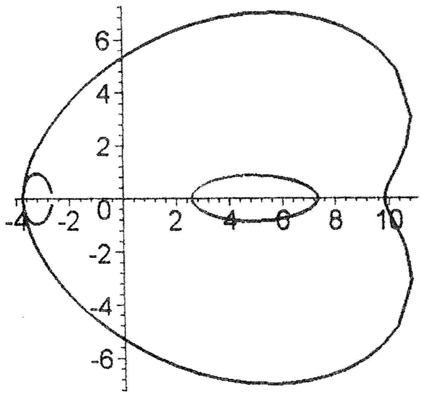


Fig 5. Defining-composion of extended oval



$$x1 = 1,\ x2 = 2,\ x3 = 3,\ x4 = 5,\ x5 = \frac{150}{23},\ x6 = \frac{165}{19}$$

> (x1=1,x2=2,x3=3,x4=4,x5=6)

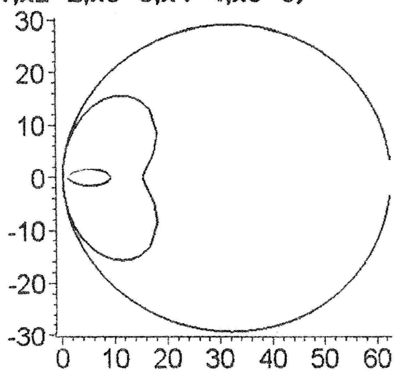


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Fig. 6 generalized Oval (chocoid)

As shown In fig. 5, it is shown by EBISUI (2001) that the process of the extension contained the existance of 6 focus points. Now, we only show extended curve in fig. 6, using generalized orthopole. In fig. 6, we show the curves express multi closed curve, and show coordinates of focus points. x1, x2, ..,x6 means F1, F2, ...,F6

4. Extension of the oval line using Simson lines

Now an extension method of the oval line is explained and this corresponds to the definition of a combination of two Simson lines from points of both ends in the diameter. Moreover, the extension of Simson lines is described in detail by Kiyomiya teacher.

4.1 how to draw the generalized oval lines using two Simson lines at the special position.

Now O, F1, F2, F3, F4 and F5 are fixed as fixed points on the straight line g and h1, h2, h3, h4 and h5 lines perpendicular to g from F1, F2, F3, F4 and F5 are drawn as axis. Here the moving point T is determined in the right side of F5. And a circle with diameter of OT is drawn out. Triangle, quadrangle and pentagon which are adapted to Simson lines are formed from intersection points of h1, h2, h3, h4 and h5. Two Simson lines related with triangle, quadrangle and pentagon from two points of O and T intersect perpendicularly, in parallel and then perpendicularly, respectively as shown in fig. 7.
It was stated in second paragraph that this intersection point related with triangle draw the oval line if T moves. Here the trajectory of points intersected perpendicularly related with pentagon (five axes) is considered. The trajectory is one of fifth power of 2, in other words 32 points. This called the generalization of the oval line using Simson lines and each curve of 32 points is called Tajicoid. As specification of coordinates of intersection points for making Tajicoid CG, as shown in fig. 8, foot points of perpendicular lines to straight lines connecting 5+5 points on the circle and other foot points were

labeled.

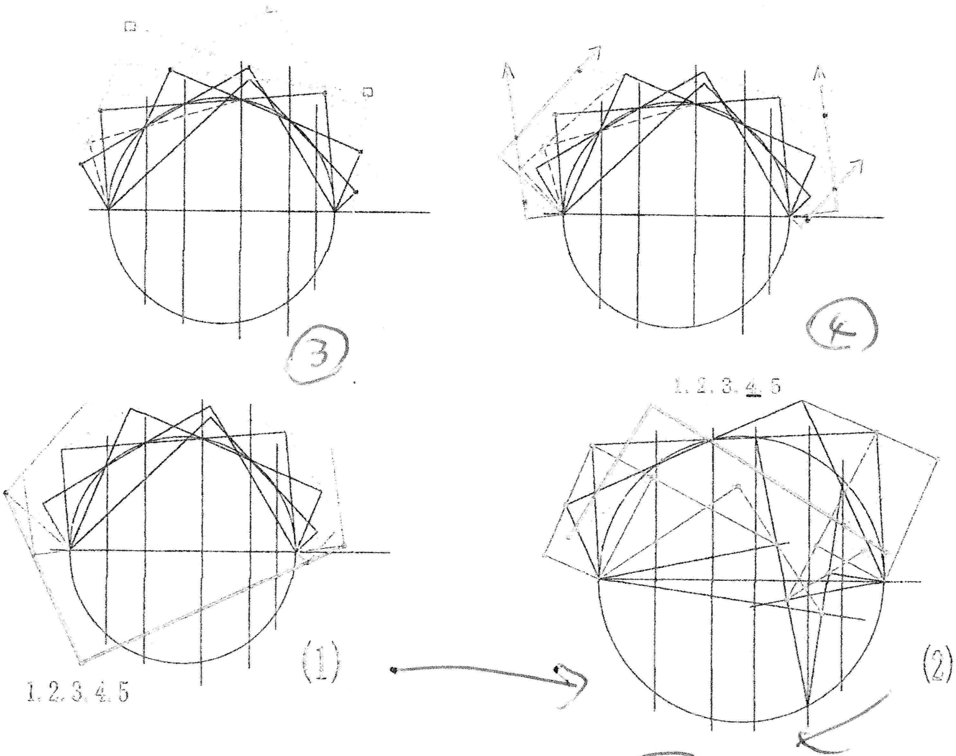

Fig. 7 Extensional property of Simson lines which define the oval line

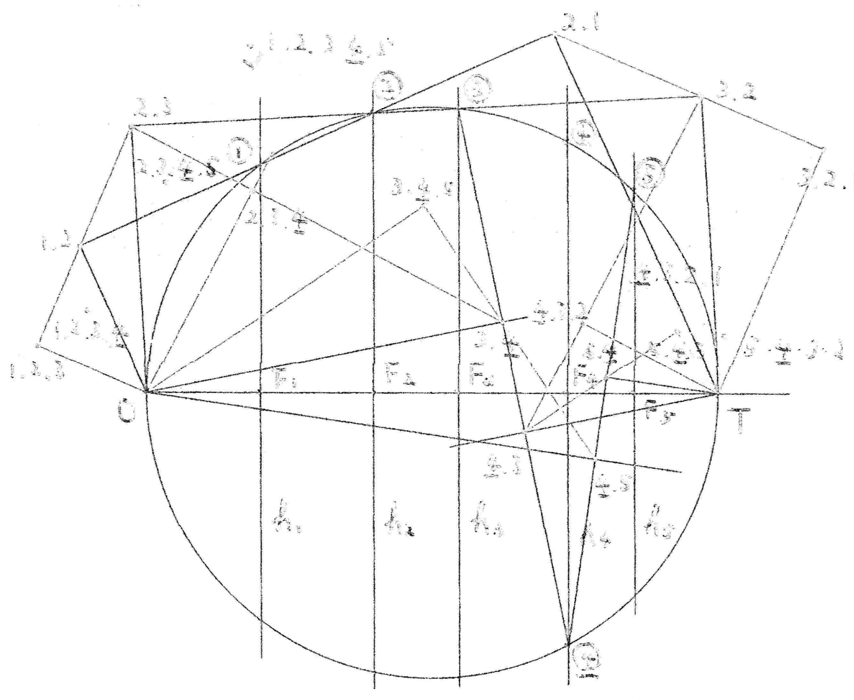

Fig. 8 labeling coordinates in drawing for defining Tajicod

4.2 CG of Tajicoid

The generalized form (Tajicoid) which extended the oval is shown in fig. 9. This CG program cannot draw yet the whole curve because parameters are not changed up to infinity. But, we can see simular forms of the oval.

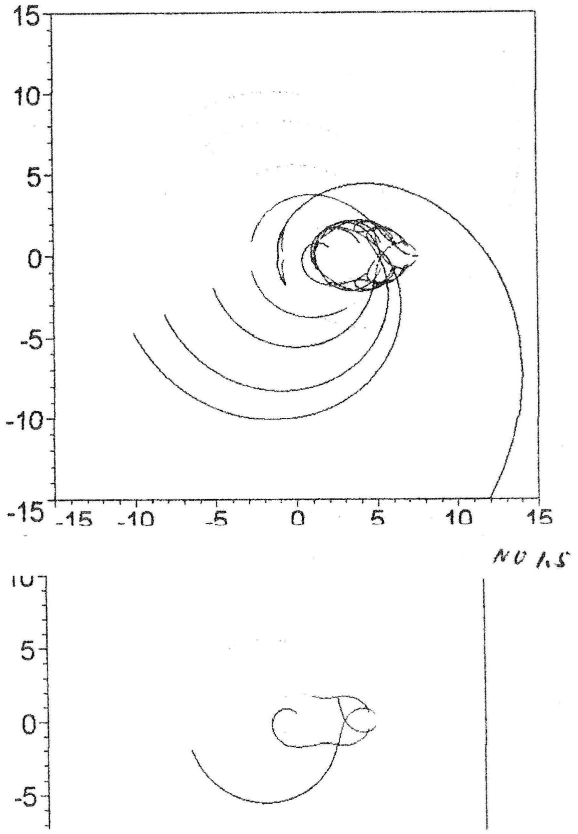

Fig. 9 generalized curve of the oval line(Tajicoid)

5. Summary

As mentioned above, in the generalized curve of the oval line described here, only appropriate CG is obtained, but simplification of the structure is not yet achieved still Algebraic, analytical, geometrical, topological research and research on formula manipulation have to be accomplished in future. We want to examplify the generalization of the oval line as an agenda of efforts for endless pursuing science by mankind and the common use.

We don't think also that the elementary geometrical definition in generalization of the oval line matches present mathematical science. However this may occur because the structure of circle, ellipsoid and oval line does not reach the level of sequence structure, algebraic structure and topological structure in mathematical structures. Anyway, we can also define infinite chain of the generalized curve of the oval. And other extensions of the oval are shown by Ebisui(1998,2000). We believe that in curves like Tajicoid , the harmony of close curve with opening curve is one step for one of the absolute with the relative in spacious structure and this belongs to the harmony of agendas among mathematical and philosophical agenda of the infinity and the finite.

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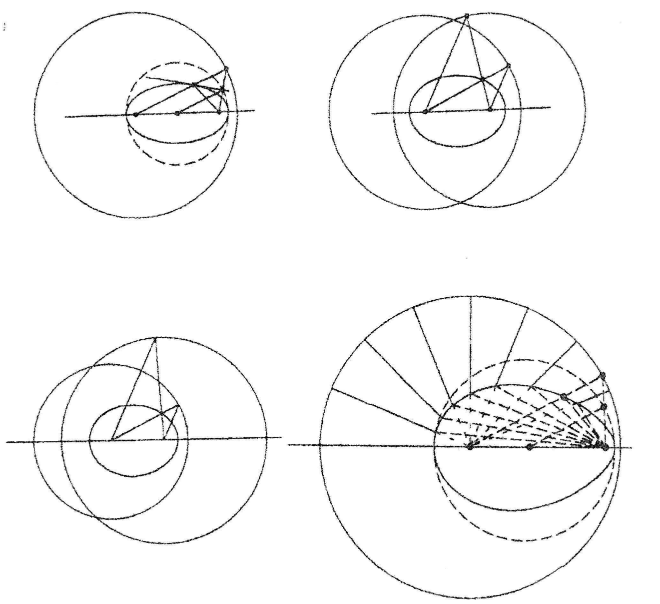

Subsidiary figure elementary constitutional drawing of ellipsoid and the oval line