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About the Oval (Doval)

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ABSTRACT: In this paper, we mention about the oval and it's new name (Doval). First, we explain the reason of naming the Oval as Doval. Next, we explain it's Definitions and Some properties and theorems, those are old and new results. And more, we describe the confocal curve of Extended Doval (chocoid), this Extended curves are defined using Orthopole theorem and they are drawn by Maple Soft. At Last, moreover we show the figure of one Extended Doval curves (Tajicoid). We append unsolved problems of Doval. We must appreciate Rapid CAD and Maple developers.

Keywords: the Doval , confocal, chocoid, Tajicoid,Geometry, Maple, Photron Rapid cad

1. INTRODUCTION

We are familiar with closed curves. But, we are not so familiar with double closed curves. Here, we define the Doval as two closed curves. Inner curve (part) is always Oval or convex. This is the reason that Doval is called as Oval. But Outer curve is not always convex. Its condition is $e_R+e_L<1$, then it's convex (Fig.1). We use a curvature of vertex to proof the problem.[1)] We can find many methods of definition or draw-theorem of Doval. About two of them should be memorized. This reason is that they are elemental and essential. And, we can define Doval, but, more over, we must study the properties of Doval. <u>This time, we mention some composition, structure, expression, and theorem on Doval.</u>
These concepts have already been studied but, we want to summarize here about them.

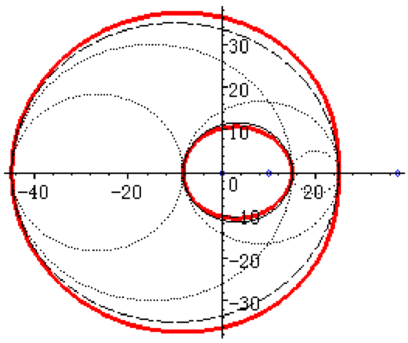


Fig.1 Doval(both convex)

2. Definitions and Theorems of Doval

2.1 Definition1.

We fix two circles, and one parallel line that passes through the two centers. And, we set two cross points on the parallel line and alternative circle, and connect the cross point and alternative center. Then two radius are made and its cross point appears. This point draws while the parallel line make on turn on a center, where two circle size are same, then Ellipse appears(Fig.2) and not same, then Oval appears. So this Oval can be called as pure extension of Ellipse. If we inspect precisely that composition, then in later case, two cross points appear, and they draw inner and outer part, namely double closed curves (Doval)(Fig.3) can be drawn.[2)]

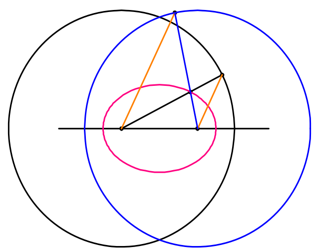


Fig.2 Ellipse(using 2 circles)

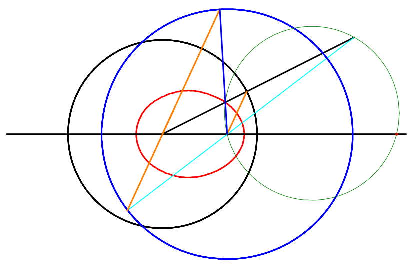


Fig.3 Doval(using 2 circles)

2.2 Def – theorem of Doval using two circles

We fix two circles (One include the other one), and can find two similar points of them. And, we can draw one parallel lines those pass through the two similar points. Then, we can obtain 8 cross points among two circles and one parallel lines. Now, we chose 2 pair points among 4 points in above 8 points, and, next, we connect the two pair points, and determine two lines. Then they are orthogonal, and make one cross point. In Above situation, four cross points appear. (See Fig. 4) .And two of them draw inner part of Doval., the remainder draw outer part, when one parallel lines make one turn on two similar points . In this situation, Figure keeps same compositions. This proof is done.[3)]

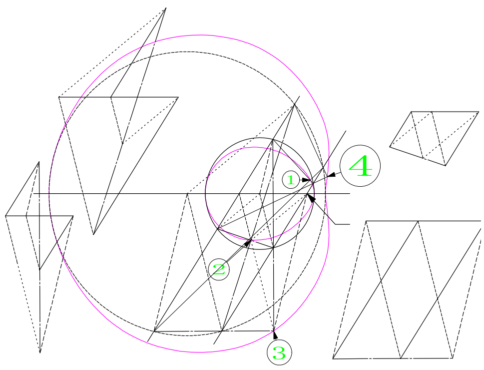


Fig.4. Doval defined by two auxiliary circles

(4 Surrounding compositions of Pappus theorem help this proof)

2.3 Theorems of Doval

2.3.1 Theorem of 3 fuci

(1) 6circles method of finding 3 foci.

One center line of one circle, we draw 3 tangent circles, and combine 2circles and draw tangent circle of two, then, more 2 circles appear. Then, totally, there are 5circles in given one circle.

In Fig.5, center circle and contour circle give Def theorem of Doval in section 2.2. Moreover, other two pairs of circle define similar compositions of the same Doval

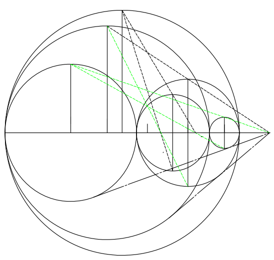


Fig.5. Three foci defined by 6 circles

2.3.2 Theorem of 4axes on Doval.

Ellipse has minor axis. And 10 years ago, we found minor axis of the oval like Ellipse. And after, we find 4 axes of Doval.

[Theorem] Si,Ai,So,Ao satisfy following Invariant equation

$$\left(\frac{Ai}{Si}\right)^2 + \left(\frac{Ao}{So}\right)^2 = 2$$

which is free from definition-circles size.,

where Si is the length of symmetry inner major axis.

Ai is the length of asymmetry inner minor axis.

So, Ao are outer cases as same as inner part.

We must pay attention that minor and major are reverse on inner and outer parts of Doval. Si, Ai, So, Ao are 4 radii of tangent circle of Doval.

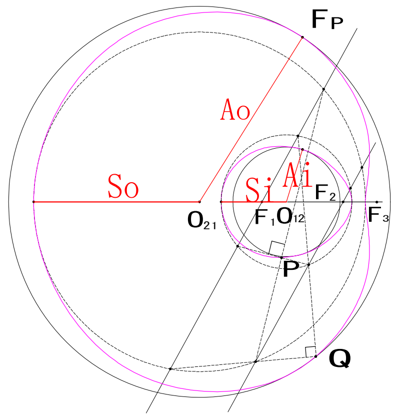


Fig.6 Relations of 4 Axes So, Ao, Si, Ai

2.4. A standard form of Doval equation.

Doval is defined by bipolar coordinates equation.

$$mr_1 \pm nr_2 = kc$$

This bipolar equation can be transformed to a standard equation by x-y coordinates

$$(m^2-n^2)^2\{y^2+X^2-\frac{(k^2m^2+k^2n^2+m^2n^2)c^2}{(m^2-n^2)^2}\}^2 = -\frac{8k^2m^2n^2c^3}{m^2-n^2}X + \frac{4k^2m^2n^2(k^2+m^2+n^2)c^4}{(m^2-n^2)^2}$$

$$X = x + \frac{n^2c}{m^2-n^2}$$

2.5. Other some properties of Doval

In this section, we mention some properties or theorems without proofs.

2.5.1. Right and Left Eccentricity determines a shape of the Oval (Doval). One Doval has (ER,EL) for inner part and (ER,－ EL) for outer part.

2.5.2. The end point of Minor Axis is not the Vertex of the Oval. This means that the end point have a special Differential Geometry meaning.

2.5.3. Perpendicular Bisectors of Asymmetry Axes pass through the 3rd focus point. This theorem applies
a definition of the third focus point-position.
2.5.4. We can not approximate All of EGGs Shape by the Ovals. Dr G.F.NAGY in Hungary and I find this result. Namely, bird eggs form are more variety than Dovals.
2.5.5. Confocal Dovals exsist. More precisely, we can say that Dovals have any two of three foci as confocal points, and 3foci as confocal points.

3. EXTENDED CURVES OF DOVAL

So for, we consider about own properties of Doval. But, we can extend Doval to hyper curves with same structure.

3.1 Chocoid

This curve Chocoid is one extension of Doval with more than 3 foci.. To define this curve, we use following composition. In Fig.7 ,we define extended Orthopole Q using fixed points O , F1, F2, F3, F4, F5, F6 on line g , and fixed perpendicular lines, h1, h2, h3, h4, h5, h6, and moving circle OT. When T moves from point F3 to infinity position on line g, Q draw one part of chocoid.

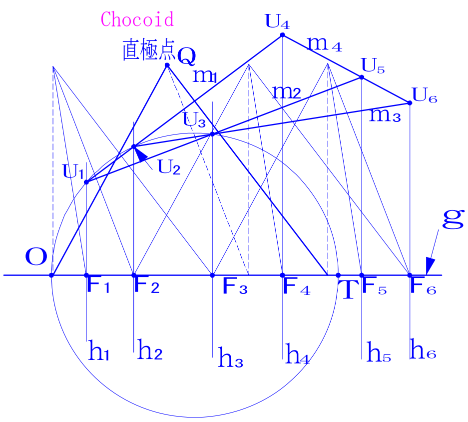


Fig.7. a definition composition of chocoid.

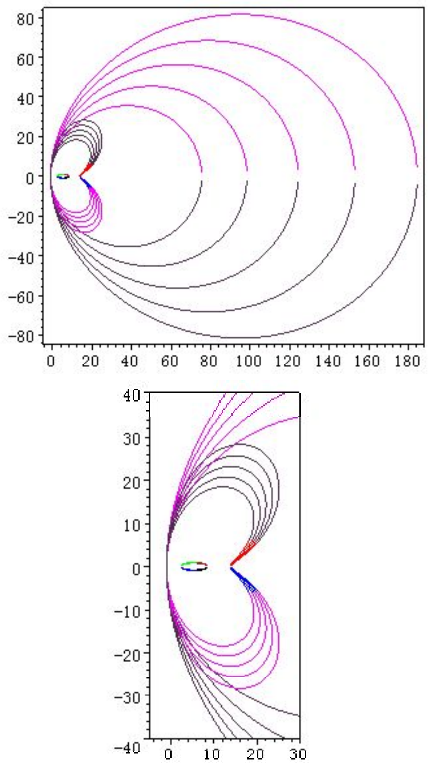
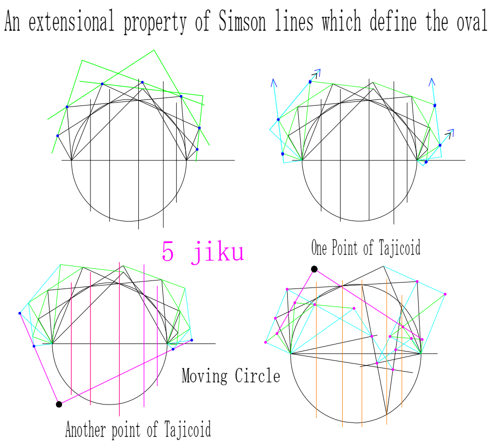
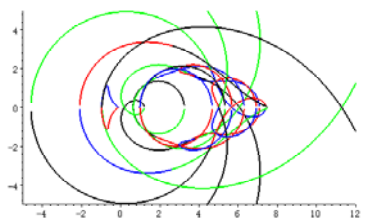


Fig.8. confocal chocoids (lowwer is precise view of upper figure)

3.2 Tajicoid

In last ICGG, we report about Tajicoid. In this paper, we show that figure and some Tajicoids with different parameters, namely different foci.





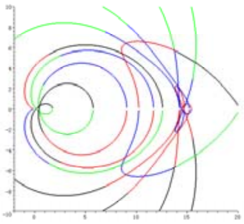


Fig.9 Def figure and Two kinds of Tajicoid (INSIDE VIEW)

We show a program of tajicoid with Parameter x1, x2,x3,x4,x5.

```
> # tajicoid-no1.5-2.5+2004-3-11 by H.E:
> #x=0-kitenn,shouten x1,x2,x3,x4,x5
> #(X1,Y1) to (X2,Y2) wo tooru Line he (0,0) yori
kudasita suisen no asi (XP,YP):
> restart:
> with(plots):
XP:=(Y1*X2-X1*Y2)*(Y1-Y2)/((X1-X2)^2+(Y1-Y2)^2):
YP:=(X1*Y2-Y1*X2)*(X1-X2)/((X1-X2)^2+(Y1-Y2)^2):
> qx12:=subs(X1=x1,Y1=y1,X2=x2,Y2=y2,XP):
> qy12:=subs(X1=x1,Y1=y1,X2=x2,Y2=y2,YP):
> qx23:=subs(X1=x2,Y1=y2,X2=x3,Y2=y3,XP):
> qy23:=subs(X1=x2,Y1=y2,X2=x3,Y2=y3,YP):
> qx34:=subs(X1=x3,Y1=y3,X2=x4,Y2=y4,XP):
> qy34:=subs(X1=x3,Y1=y3,X2=x4,Y2=y4,YP):
> qx45:=subs(X1=x4,Y1=y4,X2=x5,Y2=y5,XP):
> qy45:=subs(X1=x4,Y1=y4,X2=x5,Y2=y5,YP):
>
> rx12:=subs(X1=qx12,Y1=qy12,X2=qx23,Y2=qy23,XP):
> ry12:=subs(X1=qx12,Y1=qy12,X2=qx23,Y2=qy23,YP):
> rx23:=subs(X1=qx23,Y1=qy23,X2=qx34,Y2=qy34,XP):
> ry23:=subs(X1=qx23,Y1=qy23,X2=qx34,Y2=qy34,YP):
> rx34:=subs(X1=qx34,Y1=qy34,X2=qx45,Y2=qy45,XP):
> ry34:=subs(X1=qx34,Y1=qy34,X2=qx45,Y2=qy45,YP):
>
> sx12:=subs(X1=rx12,Y1=ry12,X2=rx23,Y2=ry23,XP):
> sy12:=subs(X1=rx12,Y1=ry12,X2=rx23,Y2=ry23,YP):
> sx23:=subs(X1=rx23,Y1=ry23,X2=rx34,Y2=ry34,XP):
> sy23:=subs(X1=rx23,Y1=ry23,X2=rx34,Y2=ry34,YP):
>
> # (X1,Y1) to (X2,Y2) wo tooru Line he (XS,0) yori
kudasita suisen no asi (XP,YP):!
s:=(-X1*X2+X1^2+Y1^2-Y1*Y2+XS*(X2-X1))/((X1-X2)^2+
(Y1-Y2)^2):
> XP:=s*(X2-X1)+X1:
> YP:=s*(Y2-Y1)+Y1:
>
> qx21:=subs(X1=x1,Y1=y1,X2=x2,Y2=y2,XP):
> qy21:=subs(X1=x1,Y1=y1,X2=x2,Y2=y2,YP):
> qx32:=subs(X1=x2,Y1=y2,X2=x3,Y2=y3,XP):
> qy32:=subs(X1=x2,Y1=y2,X2=x3,Y2=y3,YP):
> qx43:=subs(X1=x3,Y1=y3,X2=x4,Y2=y4,XP):
> qy43:=subs(X1=x3,Y1=y3,X2=x4,Y2=y4,YP):
> qx54:=subs(X1=x4,Y1=y4,X2=x5,Y2=y5,XP):
> qy54:=subs(X1=x4,Y1=y4,X2=x5,Y2=y5,YP):
>
> rx21:=subs(X1=qx21,Y1=qy21,X2=qx32,Y2=qy32,XP):
> ry21:=subs(X1=qx21,Y1=qy21,X2=qx32,Y2=qy32,YP):
> rx32:=subs(X1=qx32,Y1=qy32,X2=qx43,Y2=qy43,XP):
> ry32:=subs(X1=qx32,Y1=qy32,X2=qx43,Y2=qy43,YP):
> rx43:=subs(X1=qx43,Y1=qy43,X2=qx54,Y2=qy54,XP):
> ry43:=subs(X1=qx43,Y1=qy43,X2=qx54,Y2=qy54,YP):
>
> sx21:=subs(X1=rx21,Y1=ry21,X2=rx32,Y2=ry32,XP):
> sy21:=subs(X1=rx21,Y1=ry21,X2=rx32,Y2=ry32,YP):
> sx32:=subs(X1=rx32,Y1=ry32,X2=rx43,Y2=ry43,XP):
> sy32:=subs(X1=rx32,Y1=ry32,X2=rx43,Y2=ry43,YP):
>
> # (sx12,sy12)-(sx23,sy23)=line  kouten(XK,YK)
(sx21,sy21)-(sx32,sy32)=line:
>
XK:=-((sx12*sy23-sy12*sx23)*(sx21-sx32)-(sx21*sy32
-sx32*sy21)*(sx12-sx23))/((sy12-sy23)*(sx21-sx32)-
(sy21-sy32)*(sx12-sx23)):
YK:=((sy12-sy23)*(sx21*sy32-sx32*sy21)-(sy21-sy32)
*(sx12*sy23-sx23*sy12))/((sy12-sy23)*(sx21-sx32)-
(sy21-sy32)*(sx12-sx23)):
> j:=0:
> colorpared:=[black,red,blue,green]:
> for i1 from -1 to 1 by 2 do
                                    for i2 from -1 to
1 by 2 do
                                 for i3 from -1 to 1
by 2 do
                         for i4 from -1 to 1 by 2 do

            for i5 from -1 to 1 by 2 do  j:=j+1:
XD:=subs(XS=t,x1=1.5,y1=i1*sqrt(1.5*t-1.5^2),x2=2.
5,y2=i2*sqrt(2.5*t-2.5^2),x3=3,y3=i3*sqrt(3*t-3^2),
x4=4,y4=i4*sqrt(4*t-4^2),x5=5,y5=i5*sqrt(5*t-5^2),
XK):
YD:=subs(XS=t,x1=1.5,y1=i1*sqrt(1.5*t-1.5^2),x2=2.
5,y2=i2*sqrt(2.5*t-2.5^2),x3=3,y3=i3*sqrt(3*t-3^2),
x4=4,y4=i4*sqrt(4*t-4^2),x5=5,y5=i5*sqrt(5*t-5^2),
YK):                  T[j]:=plot([
XD,YD,t=5..infinity],view=[-15..15,-15..15],numpoi
nts=100,color=colorpared[(i4+3)+(i5+3)/2-2]):
```

```
od;od;od;od;od;
> display ({seq(T[j],j=1..32)});# by H.E:
```

4. CONCLUSION

So far, we mention about Doval and its extensions. Tajicoid, and chocoid can be extended to higher chained curves. But, now, recent PC must need more ability in CPU speed and memory and Maple Soft Technique. If it can be done, Higher chained Tajicoid and chocoid can show more interesting forms.

We have a lot of unsolved questions about Doval. For Example

1. How do more than two eccentricity exit in extended Doval?
2. What is the eccentric angle of Doval?

Ellipse is defined using x=a*cos(s), y=b*sin(s). In this formula, s is called as the eccentric angle.

5. REFERENCES

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