Abstract for 9th ICGG

To Mr.V.O.Thomas / Dpt.Of Mechanical &Manufacturing Engineering, RMIT University, Bundoora 3083, Victoria, AUSTRALIA
Fax +61-3-99256108

From Hirotaka Ebisui
Motomachi 4-12-10
Iwakunishi 740-0012 Japan
Tel +81-827-22-2573
Fax +81-827-22-3305
Email hirotaka.ebisui@nifty.ne.jp

On asymmetry axes and invariant of The oval of Descartes

Abstract :
We define the inner(+) and the outer(-) part of the Cartesian Oval by the equation mr1 ± nr2=kc on bipolar coordinates.(if m=n, then it is ellipse or hyperbola.) From this definition and some other properties, the Cartesian Oval is the pure extension of the conic section with the same structure.

In former days, We could consider Minor axis(asymmetry axis) of the inner part of the Oval, and using the same method, we can define Major axis(asymmetry axis) of the outer part of the Oval. This major axis is a segment which connects the middle point O of symmetry axis and the point Fp on the oval,which is at the longest distance from the point O. Then, the length of major axis is ao*(1+eL*eR)^(1/2), where ao is a half of the length of the symmetry-axis, and eL(resp. eR) are left (resp. right) eccentricity of the Oval. The point Fp on the outer part of the oval is equally distant from two poles(foci). The line OFp is the normal line at the point Fp of the Oval. The proof and these properties of major axis are the same as the minor axis. Next, we show that the perpendicular bisectors of inner minor axis and outer major axis pass through the third focus point. And, we can say that Cardioid (r = ao*(1-cos θ)) is the special case of Cartesian Oval. In this case, eL and eR are equal to 1 and the length of the major axis is ao*2^(1/2). Moreover, we have found the following Theorem.

[Theorem] Let bi be the length of Minor axis of the inner part of the Oval, let ai and ao be the half length of symmetry-axis of the inner and outer part, respectively. Let bo be the length of the Major axis of the outer part. Then, the following invariant holds. (bi/ai)^2+(bo/ao)^2=2

Keywords : Oval, Outer part,outer Major axis, inner Minor axis, Invariant